

Probing the Planck Scale with Neutrino Oscillations

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Quantum gravity “foam”, among its various generic Lorentz non-invariant effects, would cause neutrino mixing. It is shown here that, if the foam is manifested as a nonrenormalizable effect at scale M , the oscillation length generically decreases with energy E as $(E/M)^{-2}$. Neutrino observatories and long-baseline experiments should have therefore already observed foam-induced oscillations, even if M is as high as the Planck energy scale. The null results, which can be further strengthened by better analysis of current data and future experiments, can be taken as experimental evidence that Lorentz invariance is fully preserved at the Planck scale, as is the case in critical string theory.

The noisy vacuum of a quantum theory of gravity could a priori be imagined to have a variety of effects on the wavefunction of a particle traveling through it. Although unitarity would probably constrain the effects of the vacuum on the wavefunction’s amplitude, one could imagine that its phase might be shifted by the local effects of quantum black holes and the like which quickly pop in and out of the vacuum. Also, the type (flavor) of the particle might be affected: for example, a virtual or quantum black hole could as Hawking has suggested [1], erase information by swallowing a neutrino of one type and spit out one of a different type as it evaporates. Thus, possible effects that could be considered include Lorentz invariance violating (LIV) indices of refraction [2, 3, 4], flavor oscillations of neutral particles[5, 6, 7, 8, 9], and corrugations of wavefronts, i.e. Rayleigh scattering [8]. Rayleigh scattering, which would imply momentum non-conservation, appears to be ruled out by the observation that 10 MeV neutrinos propagated without Rayleigh scattering from supernova 1987A [8] (assuming that their scattering cross-section off cells of Planck foam scales as E^2). We conclude that if such “foam” effects exist, they must preserve translation invariance on the Planck distance scale and on larger ones; thus any non-vanishing effects must be cooperative and coherent over space-like distances in this range of scales; it may cause wave dispersion but not opalescence or translucence.

There has been considerable work on dispersive, LIV vacua. Colladay and Kostelecky have exhaustively classified renormalizable LIV effects [10, 11]. Coleman and Glashow [6, 7] discuss neutrino oscillation from renormalizable LIV terms, which yield an oscillation length that scales as $1/E$, and their results imply that any such effect that is of order unity at the Planck scale is already ruled out by many orders of magnitude. Amelino-Camelia et al. considered a photon velocity that has a linear term proportional to E/M and showed that it is marginally consistent for astrophysical gamma ray pulses if M is of order 10^{16} GeV.

The leading quantum gravity theory - critical string theory - predicts that in flat (empty) space all interactions are Lorentz invariant (LI) down to, and including, the Planck scale. LI in flat space is crucial to the internal mathematical consistency of string theory in that it guarantees that its symmetries are valid at the quantum level (see, for example, [12]). However, the Universe, possessing a preferred frame of reference, is clearly not LI. String theory allows such spontaneous breaking of LI, by allowing the possibility that the curvature and additional moduli fields have time- and space-dependent expectation values, but it does not allow LIV Planck scale “foam”.

In this paper we consider a particular class of LIV effects - *nonrenormalizable* effects of the vacuum, which could be induced by a high energy/short distance physical cutoff - and show that they give rise to a neutrino dispersion that is linear in E/M . This is the dispersion relation considered by Amelino-Camelia et al. [2] for photons, but, like Coleman and Glashow[6, 7], we consider neutrino oscillations that it could induce. In contrast to the latter, we consider nonrenormalizable corrections, which depend on a higher power of E/M . If the difference between the propagation velocities of different neutrino flavors is proportional to E/M then the oscillation length is proportional to $(E/M)^{-2}$. (Amusingly, for $M \sim M_{\text{Planck}}$ it would give an oscillation length of order an astronomical unit for solar neutrino energies and atmospheric length scales for atmospheric neutrino energies[8], but see below.) We then apply the argument[8] that high energy experiments of neutrino oscillations with E^{-2} mixing lengths should be able to “detect” nonrenormalizable cutoffs as high as the Planck scale. We argue that such experiments can, by not detecting generic effects of such cutoffs, provide experimental support for critical string theory versus many proposed alternative theories, as the latter imply LIV physical cutoffs, while critical string theory provides a LI cutoff.

It is clear that our arguments are generic, and restrict or rule out only those models of

quantum foam in which neutrino flavor is unprotected by a symmetry and Lorentz invariance is broken by nonrenormalizable interaction terms, as pointed out in [13, 14]. It remains to be seen whether any specific models of quantum gravity effects can evade our conclusions, as suggested in [13, 14].

To understand the appearance of nonrenormalizable LIV terms we model any type of short distance interaction which may be induced by quantum gravity foam, black holes swallowing particles and spitting them out, ultraviolet LIV cutoff physics, etc. First, we assume that such interaction is strong only at some characteristic energy scale M (e.g. M_{Planck}) so it can be modeled by the following interaction term in the effective Lagrangian,

$$\mathcal{L}_{\text{int}} = M \int d^4x' \psi_A^{\dagger\rho}(x) f_{\rho\sigma}^{AB} [M(x-x')] \psi_B^\sigma(x'). \quad (1)$$

Here f determines the interaction strength, and is therefore assumed to be small when its argument is larger than unity, $A, B = 1, 2$ label different neutrino eigenstates of the total Hamiltonian, $\rho, \sigma = 1, \dots, 4$ are fermionic indices. To avoid Rayleigh scattering as discussed previously we have assumed that f is a function only of the distance four-vector $x - x'$. We may now expand $\psi_B^\sigma(x') = \psi_B^\sigma(x) + \partial_\mu \psi_B^\sigma(x)(x' - x)^\mu + \frac{1}{2} \partial_\mu \partial_\nu \psi_B^\sigma(x)(x' - x)^\mu (x' - x)^\nu + \dots$, where the dots stand for higher derivative terms. This expansion may be used to convert the interaction Lagrangian (1) into a series of local terms. Each additional derivative comes with an additional power of $1/M$, demonstrating that the terms induced by (1) indeed depend on powers of E/M . The lowest dimension nonrenormalizable terms are dimension five operators and therefore depend on E^2/M . They are given by the second moments of $f_{\rho\sigma}^{AB}$. It may well be that the renormalizable terms which could have been induced by (1) are absent. For example, if $f_{\rho\sigma}^{AB}$ depends only on the absolute value $|x - x'|$, no dimension four operators are induced. If, in addition, neutrino masses are protected, say, by chiral symmetry, and $f_{\rho\sigma}^{AB}(|x - x'|)$ factors into constant matrix $M_{\rho\sigma}^{AB}$ and a universal kernel $f(x - x')$, then neither are dimension three operators induced.

Consider the diagram in Fig. 1. The effective interaction vertex connecting the incoming and outgoing states includes, by assumption, quantum gravity effects. Because any flavor gravitates, the effective vertex does not generically respect the global flavor symmetry (e.g. a quantum black hole swallowing one type of neutrino and evaporating into another type) so in its presence the eigenstates of the weak interactions $|\alpha\rangle, |\beta\rangle$ are mixtures of the eigenstates of total Hamiltonian $|A\rangle, |B\rangle$, $|\alpha\rangle = \cos\theta|A\rangle + \sin\theta|B\rangle$, and $|\beta\rangle = -\sin\theta|A\rangle + \cos\theta|B\rangle$,

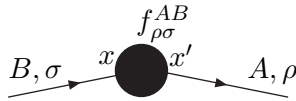


FIG. 1: Effective interaction vertex induced by quantum gravity effects.

and therefore the interaction terms obtained from (1) induce mixing between flavor states. We will argue shortly that the induced mixing angles are generically large $\sin^2(2\theta) \sim 1$. (In [13] it is claimed that in certain “kinematical models” flavor mixing does not arise).

The possibility of neutrino oscillations arises, in general, when terms are added to the neutrino sector of the Standard Model Lagrangian, such that the eigenstates of the total Hamiltonian are not coincident with, but rather linear combinations of the different neutrino flavors. In the simplified case of two flavors, and a single oscillation inducing interaction, a neutrino of flavor α can oscillate into a neutrino of flavor β after having traveled for a distance X , with a probability that is usually expressed in terms of a mixing angle θ and an oscillation length L

$$P_{\alpha \rightarrow \beta} = \sin^2(2\theta) \sin^2 \left[\frac{\pi X}{L} \right]. \quad (2)$$

The oscillation length $L = \frac{2\pi}{E_A - E_B}$, is defined in terms of the difference of the states’ energy $E_A - E_B$ and does not depend (for this case) on the mixing angle θ . The mixing angle determines only the amplitude of the oscillations. In the case of more than two neutrino flavors, eq. (2) becomes more complicated, and includes more than one oscillatory term. Since eq. (2) contains the essential physical ingredients of the oscillation phenomenon, and is much simpler than the corresponding expression in the general case, we will concentrate on the two flavor case.

If oscillations are induced by more than one interaction, L should be replaced by the total oscillation length L_{tot} , which is given by a combination of the individual oscillation lengths L_n that each individual interaction would have induced on its own. In the two flavor case (θ_n are the mixing angles of each single effect) one obtains [7]

$$L_{tot}^{-1} = \frac{\sum_n L_n^{-1} \cos 2\theta_n}{\cos 2\theta_{tot}}, \quad (3)$$

from which it is clear that in general the total oscillation length is dominated by the shortest one. This feature remains valid also for the case of more than two flavors, though eq. (3) is

replaced by a more complicated expression.

Thus, if we are interested in a case in which one oscillation inducing term (say L_j) dominates, we may replace L by L_j instead of using $L = L_{\text{tot}}$ in eq. (2), and use the factorization property of eq.(2) which implies that the oscillation length does not depend on the mixing angles.

The most studied oscillation mechanism is due to the presence of neutrino mass. For this case, LI implies that the oscillation length for ultrarelativistic neutrinos is given by $L \sim \frac{4\pi E}{\Delta m^2}$, Δm^2 being the neutrino squared mass difference and here $c = 1 = \hbar$. In fact, every LI term in the effective action will give the same oscillation length energy dependence as a mass term. On the other hand, energy dependence different from $L \propto E$ is a smoking-gun signal of violation of LI. Several examples of LIV effects have appeared in the literature: $L \propto E^0$ [6, 9], $L \propto E^{-1}$ [7, 15] and also $L \propto E^{-2}$ [8, 16] and $L \propto E^{-3}$ [17].

Recent SuperKamiokande (SK) results indicate that a μ - τ mass mixing mechanism is the best candidate for explaining the observed μ - e anomaly in the atmospheric neutrino flux[18]. The analysis of energy dependence of SK data excludes LIV effects as the primary source for the observed μ - e anomaly. Furthermore, by analyzing models in which LIV terms are present in addition to the mass mixing, it is possible to put stringent upper limits on their strength[19]. We show below that existing experimental data are already sufficient to provide important clues about the nature of neutrino interaction at the Planck scale, and that with better analysis and with more data LIV terms could be ruled out to high accuracy. Our claim is based on a preliminary analysis of the $L \propto E^{-2}$ case, which are induced only by nonrenormalizable terms in the neutrino effective Lagrangian. Dimensional analysis shows that renormalizable terms in the neutrino effective action induce oscillation lengths proportional to E^{-1} , or to a non-negative power of E . In particular, all possible renormalizable and rotation invariant mixing terms in the neutrino effective action induce oscillation lengths proportional either to E^0 , to E , or to E^{-1} . This can be checked by a case-by-case analysis of the dispersion relation induced by all the operators that are bilinear in the neutrino field, and have mass dimension not exceeding four. A complete set of such operators is $\bar{\psi}\gamma^0\psi$, $\bar{\psi}\gamma^5\gamma^0\psi$, $\bar{\psi}\partial_t\psi$, $\bar{\psi}\gamma^5\partial_t\psi$, $\bar{\psi}\gamma^0\partial_t\psi$, $\bar{\psi}\gamma^5\gamma^0\partial_t\psi$, $\bar{\psi}\gamma^{0i}\partial_i\psi$, $\bar{\psi}\gamma^{ij}\partial^k\epsilon_{ijk}\psi$. (We have not included in this list operators that can be obtained as linear combinations of them and LI terms). It follows that only terms that are both nonrenormalizable and LIV can be the origin of $L \propto E^{-2}$.

We now wish to show that generic mixing angles that are induced are large. Assuming that gravity is “flavor blind” the interaction Hamiltonian induced by the terms in eq.(1), expressed in flavor basis is a “democratic” matrix $H_{\text{int}} = \begin{pmatrix} g(E) & g(E) \\ g(E) & g(E) \end{pmatrix}$, and the total Hamiltonian (including possibly mass mixing) in the flavor basis could be expressed as $H_{\alpha\beta} = \begin{pmatrix} h_{11} + g(E) & h_{12} + g(E) \\ h_{21} + g(E) & h_{22} + g(E) \end{pmatrix}$, where $h_{12} = h_{21}$. In the limit that the energy difference between the Hamiltonian eigenstates $\Delta E = |E_1 - E_2|$ is dominated by $g(E)$, the Hamiltonian eigenvectors are given by $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and therefore mixing is maximal. In case $|g(E)| > |h_{12}|$, $|h_{11} - h_{22}|$ we therefore expect a large mixing angle. Since the SK (and possibly also solar neutrino) data appear to imply a large mixing angle due to LI effects, at least among some pairs of neutrino types, we infer that $h_{12} \geq |h_{11} - h_{22}|$, so that $|g(E)| > |h_{12}|$ implies $|g(E)| > |h_{11} - h_{22}|$ and is by itself sufficient to infer a large mixing angle.

As a simple explicit example of the impact of a local term resulting from the interaction Lagrangian (1), consider adding a dimension five LIV interaction term to the standard kinetic term of massless neutrinos,

$$\mathcal{L} = i\delta^{AB}\bar{\psi}_A\partial\psi_B - \frac{S^{AB}}{M}\bar{\psi}_A\partial_t\partial_t\gamma^0\psi_B, \quad (4)$$

with $S^{AB} = \text{diag}(S_A, S_B)$. To obtain S^{AB} from the second moments of $f_{\rho\sigma}^{AB}$ we have used the γ -matrices as a basis to 4 by 4 matrices. In terms of the “democratic” interaction Hamiltonian which we have discussed previously one finds $g(E) = \frac{E^2}{2M}(S_A - S_B)$.

As we have argued, an interaction term as in eq.(4) is generic to every nonrenormalizable theory that is also LIV. We will explicitly show that the resulting oscillation length satisfies $L \propto E^{-2}$. Other terms such as $\frac{1}{M}\bar{\psi}\partial_i\partial_i\gamma^0\psi$, $\frac{1}{M}\bar{\psi}\partial_t\partial_i\gamma^i\psi$, $\frac{1}{M}\bar{\psi}\partial_t\partial_t\gamma^5\gamma^0\psi$, $\frac{1}{M}\bar{\psi}\partial_i\partial_i\gamma^5\gamma^0\psi$, $\frac{1}{M}\bar{\psi}\partial_t\partial_i\gamma^5\gamma^i\psi$ also produce $L \propto E^{-2}$. Recall that in this case the oscillation length depends only on energy difference between the oscillating states, and does not depend on the mixing angle between flavor eigenstates which determines only the amplitude of oscillations.

The equation of motion for the physical eigenstates is $(p_\mu\gamma^\mu - \frac{S_{A,B}E^2}{M}\gamma^0)\psi_{A,B} = 0$; by multiplying this equation by $p_\mu\gamma^\mu - \frac{S_{A,B}E^2}{M}\gamma^0$ and using the anticommutation relations of gamma matrices, we obtain $E^2\left(1 - \frac{S_{A,B}E}{M}\right)^2 - \mathbf{p}^2 = 0$, and therefore, to first order in $S_{A,B}E/M$ $E \sim |\mathbf{p}|\left(1 + \frac{S_{A,B}E}{M}\right)$, from which it follows that

$$L \sim \frac{2\pi}{S_A - S_B} \frac{M}{E^2}, \quad (5)$$

independently of the mixing angle between flavor eigenstates.

A LIV term could also arise in an otherwise LI theory, such as critical string theory, via spontaneous breaking of Lorentz symmetry. For example, a LIV contribution of the form (4) could be generated, when a field (which could be the dilaton, or any other of the moduli fields of string theory) is slowly rolling in a background of a non-vanishing gravitational potential. But in this case, the breaking would be proportional to the time derivative of the field ($\dot{\Phi}/M$) whose magnitude is severely constrained. If $\dot{\Phi}$ is non-vanishing then Φ has kinetic energy. The requirement that the kinetic energy density of Φ is not larger than universe closure density provides an incredibly stringent bound $\dot{\Phi}/M_{\text{Planck}} < 10^{-61}$, which makes spontaneous breaking effects completely undetectable. This conclusion extends to all the terms that can produce an $L \propto E^{-2}$ behavior since all of them must contain at least one time (or space) derivative of a field, and therefore are subject to the same (or similar) phenomenological constraints.

Because all nonrenormalizable theories are expected to generate higher derivative terms at the cutoff scale, and because such terms are not necessarily LI unless the physical cutoff mechanism itself is intrinsically LI, as in critical string theory, a detection (or exclusion) of $L \propto E^{-2}$ carries with it information about the high energy/short-distance properties of the theory. In contrast, the detection of an energy dependence $L \propto E^{-1}$ produced by renormalizable terms (though in any case extremely interesting) would not carry with it any such information. If LIV at the cutoff scale is explicit instead of spontaneous, typical mixing terms are expected to be of order unity (and detectable, as we show below), simply because there is no symmetry that protects them. It follows that the detection of $L \propto E^{-2}$ effect would be a definite signal that nonrenormalizable and LIV terms are present in the neutrino effective action. We conclude that the detection of $L \propto E^{-2}$ dependence in neutrino oscillations would constitute strong evidence against critical string theory, in which the ultraviolet cutoff is realized in a LI way. Conversely, an experimental exclusion of such an effect should be considered as vindication of critical string theory as compared to other Planck scale models that violate Lorentz symmetry.

If one evaluates eq.(5), with $E \sim 1$ GeV, $M \sim M_{\text{Planck}}$, and $S_A - S_B = O(1)$ as expected in explicit LIV where $g(M) \sim M$, one obtains an oscillation length of about 10 km. Typical high end of the energy range of solar neutrino observatories is about 10 MeV which results in an oscillation length of about 10^{-3} AU. These estimates suggests that if the mixing

is close to maximal mixing $\sin^2(2\theta) = 1$, then previous neutrino oscillation experiments CHORUS[20] and NOMAD[21], and the currently operating experiments SK[18], K2K[22], and SNO[23], are already able to detect the proposed effect if $S_{A,B}$ is larger than about 10^{-2} . In this context, the existing data must be interpreted as a null result, $S_{A,B}$ less than about 10^{-2} , or very small mixing $\sin^2(2\theta) \ll 1$ or both, accordingly to equation (2), as the analysis of the energy dependence of the oscillation length of SK data shows that the observed oscillation is due to mass mixing $L \propto E$, rather than to LIV terms[19]. Planned experiments MINOS and CNGS[24] will be able to strengthen and verify these findings. The situation is summarized in table I which assumes maximal mixing. As can be seen from the table, we can already conclude without further analysis that the actual upper bound on α is about 10^{-3} for maximal mixing, and that the forthcoming experiments are potentially capable of improving this limit by at least one order of magnitude. Translating this bound on α into a bound on M , we obtain $M > 10^3 M_{\text{Planck}}$. It is worth noticing that the experimental sensitivity for other possible sources of nonrenormalizable LIV, such as the detection of time delays of photons from distant astrophysical sources, turns out to be significantly lower $M \sim M_{\text{Planck}}$ (see Table I in [25]). A more detailed analysis could enhance the sensitivity of neutrino oscillation experiments compared to the estimates in the table, as shown in Fig. 2, and determine exclusion regions in $\alpha, \sin^2(2\theta)$ plane.

Furthermore, all these experiments observe (or plan to observe) a broad spectrum of incoming neutrino energies, from about 10 MeV to few hundreds GeV or more, so in fact each experiment sets a stronger constraint on nonrenormalizable LIV terms as the high energy end of its range is exploited. In the case of SuperKamiokande the flight-distance also varies - from a few tens of Kilometers to about 10^4 Km. Thus, better analysis of available data, and additional data from planned experiments will allow the significant strengthening of the bound on $S_{A,B}$, perhaps down to 10^{-8} . Figure 2 demonstrates that the limits reported in table I can be improved by the analysis of the high-energy part of the neutrino spectra. This is in contrast to the mass mixing case for which L is increasing with energy, where the best bounds are obtained by the lowest energy tail of neutrino spectra. Up-going muons in the SuperKamiokande experiments, reaching energies of 10^3 GeV, can provide the best bound on α at about 10^{-8} , while a value of 10^{-6} can potentially be reached by MINOS and CNGS. A more detailed analysis is required to determine the highest energy at which enough data can be accumulated.

EXP.	STATUS	$\langle E \rangle$ (GeV)	L (Km)	X (Km)	α
CHORUS	closed 1997	26	10^{-2}	0.85	10^{-2}
NOMAD	closed 1999	24	10^{-2}	0.94	10^{-2}
SK	operating	1.3	10	$10\text{-}10^4$	$1\text{-}10^{-3}$
K2K	operating	1.3	10	250	10^{-2}
SNO	operating	0.008	10^5	10^8	10^{-3}
MINOS	starting 2003	15	0.1	730	10^{-4}
CNGS	starting 2005	17	0.1	732	10^{-4}

TABLE I: Shown for each experiment are its operation status, mean value of observed neutrino energy, oscillation length according to (5) (with $S_A - S_B = 1$), typical neutrino flight distance X , and the ratio $\alpha = L/X$. Parameter α can be thought of as the value of $S_A - S_B$ for which $L = X$ for each experiment, that is, the lowest value of $S_A - S_B$ to which each experiment is sensitive, assuming maximal mixing.

We conclude that existing bounds are at most barely compatible with the existence of a LIV ultraviolet cutoff at the Planck scale. Strengthening of these bounds by better analysis and additional data from future experiments would be an imminent vindication of critical string theory, and a strike against models that allow explicit breaking of the Lorentz symmetry at the cutoff scale.

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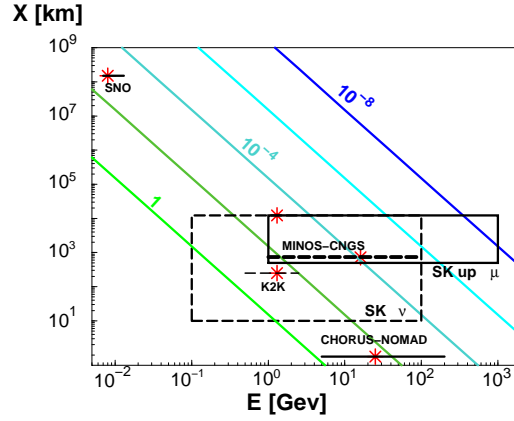


FIG. 2: Shown are neutrino energy ranges and flight distances for each experiment; the diagonal constant α lines indicate expected sensitivity. The SK reach is represented by the two rectangles: the dashed one outlines the energy range and flight distance of detected μ neutrinos, while the solid one is for up-going muons. The stars display the data reported in table I, and mark neutrino energies for which luminosity is maximal.

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 CHORUS: <http://choruswww.cern.ch/welcome.html>
 NOMAD: <http://nomadinfo.cern.ch/>
 SK: <http://www-sk.icrr.u-tokyo.ac.jp/doc/sk/index.html>
 K2K: <http://neutrino.kek.jp/>
 SNO: <http://owl.phy.queensu.ca/sno/>
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 CNGS: <http://proj-cngs.web.cern.ch/proj-cngs/cngs.htm>.
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